Name:____________________________ Math 260

Date: 4/8/2025 Exam 2

Please show ALL your work on the problems below. No more than 1 point will be given to problems if you only provide the correct answer and insufficient work.

A formula you may need: $\vec{v} \cdot \vec{w} = ||\vec{v}|| \, ||\vec{w}|| \cos \theta$

1. (10, 5, 10 points) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -2 & -1 \\ 9 & 0 & -3 \end{bmatrix}$,

a) Find $\det(A)$ using the cofactor expansion method (NO calculator)

b) Is A invertible? Why or why not?

c) Find A^{-1} using the adjoint method (No calculator)

2. (12 points) Use Cramer's Rule to solve the system of equations below. (Calculator OK)

$$7x_1 + 2x_2 + x_3 = 19$$
$$-3x_1 + 4x_2 - 2x_3 = -4$$
$$2x_1 - 5x_2 + 6x_3 = 5$$

3. (15, 5, 15, 5 points) Let $A = \begin{bmatrix} 1 & -15 & -6 \\ 0 & -4 & -2 \\ 0 & 3 & 1 \end{bmatrix}$.

a) Find the characteristic polynomial of A (No calculator)

b) Find the eigenvalues of A (No calculator)

(…this is a continuation of problem 3)

c) Find all eigenvectors of A (Calculator OK)

d) Diagonalize A by finding a diagonalizing matrix P and a diagonal matrix D such that $D = P^{-1}AP$ (No Calculator)

4. (10 points) Find the angle between the vectors $\vec{v} = (4, 2, 9, -2, 3)$ and $\vec{w} = (-3, -2, 1, 2, -5)$

5. (10 points) Find 2 different linear combinations of the vectors $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix}$.

6. (16 points) Prove that the function $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (2y - 3x, 4x)$ is a linear transformation.

7. (12 points) Prove that the function $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x + 2,\ y - 3x)$ is NOT a linear transformation.

8. (12, 12, 12 points) Prove or disprove each of the following:

a) If A and B are both $n \times n$ invertible matrices, then AB is also invertible and $(AB)^{-1} = B^{-1}A^{-1}$

b) If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^2$ and k is a scalar, then $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$

c) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$, then $\left||\vec{v} + \vec{w}|\right| = \left||\vec{v}|\right| + ||\vec{w}||$